LSMR equalization algorithm based on dynamic confidence interval detection in FBMC/OQAM systems

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ABSTRACT

For the problem of stable data reception in FBMC/OQAM systems in complex high-speed mobile environments, the traditional channel equalization algorithm has limitations in terms of computational complexity and demodulation performance. For this reason, a low-complexity equalization algorithm is proposed. First, a Least Squares Minimum Residuals (LSMR) channel equalizer for FBMC/OQAM systems is designed, and the LSMR algorithm efficiently utilizes the sparse property of the channel matrix, which significantly reduces the computation and storage requirements. In order to further improve the accuracy of signal detection, a dynamic confidence space symbol detection and interference elimination mechanism is introduced, and an iterative optimization strategy is proposed. Through the adaptive adjustment in the iterative process, the symbol judgment and interference suppression are optimized to enhance the performance of signal demodulation. Finally, the simulation results verify the effectiveness of the proposed method and ensure stable data reception in high-speed moving and multi-path interference environments.

Keywords: FBMC/OQAM, high mobility, equalization, LSMR

1. INTRODUCTION

Filter bank multicarrier (FBMC) systems have significant advantages over OFDM in terms of spectrum utilization, anti-neighbor-channel interference, and robustness of frequency-selective channels by introducing prototype filters with good time-frequency focusing characteristics[1-2]. Different from the high out-of-band emission and strict synchronization requirements of OFDM, FBMC can effectively reduce spectrum leakage, adapt to asynchronous transmission, and satisfy the requirements of low latency, high efficiency, and high reliability. However, in high-speed mobile environments, FBMC faces inter-subcarrier interference (ICI), inter-symbol interference (ISI), and inherent intrinsic interference, which exacerbate signal distortion and increase the challenges of interference cancellation and channel equalization. Therefore, optimizing the design of interference cancellation and channel equalization is a key issue to achieve efficient and stable transmission of FBMC.

For channel equalization in FBMC/OQAM systems, the traditional Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) equalization are still applicable[3]. Researchers first investigated first-order tap equalizers, which are both easy to use and highly effective. However, in most practical applications, single-tap equalizers are not sufficient. Therefore, they proposed new equalizers as well as simple interference cancellation schemes. In 2017, Nissel R et al. designed an N-tap MMSE equalizer. Compared with the traditional first-order equalizer, the N-tap equalizer can effectively suppress ISI and ICI, which enables the system to maintain a better performance in a highly dynamic environment[4]. In the same year, Nissel R et al. proposed a low-complexity interference cancellation scheme with interference reconstruction and feedback cancellation after first-order tap equalization. He Z proposed an MMSE-based Equalized Interference Cancellation (EIC) algorithm. Considering that the MMSE algorithm has a better suppression effect on noise, it was applied to the original EIC equalizer to enhance the equalization effect[5]. In 2023, Qi Y derived the phenomenon of response difference during the design of multi-tap equalizers for high frequency selective channels. To address the impact of this phenomenon on the equalization performance, a multi-stage Frequency Sampling (FS) equalizer design

International Conference on Signal Processing, Communication, and Control Systems (SPCCS 2025),
edited by Paolo Mercorelli, Yi-Zeng Hsieh, Proc. of SPIE Vol. 13705, 137051C

scheme based on MRC, ZF and MMSE criteria is proposed. The scheme operates at symbol rate with low implementation complexity[6].

In order to reduce the computational complexity of the system and improve the BER performance of equalization, this paper proposes an iterative LSMR equalization algorithm based on confidence space. The main contributions are as follows:

1. An LSMR channel equalizer adapted to FBMC/OQAM systems is designed. The LSMR algorithm efficiently utilizes the sparse characteristics of the channel matrix, significantly reduces the computation and storage requirements, and thus reduces the computational complexity while guaranteeing high accuracy.

2. In order to further improve the accuracy of signal detection, a dynamic confidence space symbol detection and interference elimination mechanism is introduced, and an iterative optimization strategy is proposed. Through the adaptive adjustment in the iterative process, the symbol judgment and interference suppression are optimized, and the performance of signal demodulation is enhanced.

3. The practical performance of the method in dual-selective channels is verified, using the 3GPP channel model as the actual channel information, and the Hermite filter is used for simulation. When the input SNR is 40dB, the BER performance of the method in dual-selective channel is close to 10^{-3}, which is better than the traditional equalization method.

2. SYSTEM MODEL

In FBMC/OQAM systems, the real and imaginary parts of the data are each sent as different real-valued symbols. $x_{l,k}$ denotes the k th symbol in the l th subcarrier and the transmit symbol can be expressed as:

$$s\left(t\right)=\sum_{l=0}^{L-1}\sum_{k\in \mathrm{Z}}x_{l,k}g_{l,k}\left(t\right)=\sum_{l=0}^{L-1}\sum_{k\in \mathrm{Z}}x_{l,k}g\left(t-\frac{nT}{2}\right)\mathrm{e}^{j\frac{2\pi lt}{T}}\mathrm{e}^{j\frac{(l+k)\pi}{2}} \tag{1}$$

where $g_{l,k}\left(t\right)$ denotes the prototype filter through which the data of the k th moment in the l th subcarrier passes, which is obtained by transforming the prototype filter function $g\left(t\right)$. The length of $g\left(t\right)$ is $L_g=KL$, where K is the overlap factor of the prototype filter, and L is the number of subcarriers of the system. T is a symbol period. After the real and imaginary parts are separated, the time-frequency resource that originally sends a complex number can send twice as much real data, which is exactly the reason why OQAM modulation brings about data rate improvement. At this time, the time interval between neighboring data $\tau_0=T/2$, and the subcarrier interval of the FBMC system is $f_0=1/T=1/2\tau_0$. However, the maximum data rate transmission comes at the cost of the system's non-orthogonality in the imaginary domain, which causes inherent interference.

In practical applications, signals are usually time-discretized. Therefore, matrix theory is used to model the time-discretized system in order to characterize the signal processing process of the FBMC/OQAM system. Setting the sampling frequency $F_s=f_0N_{FFT}$, the filter $g_{l,k}(t)$ is sampled and represented in vector form as $\mathbf{g}_{l,k}\in\mathbb{C}^{N\times 1}$. The filter vectors are stacked in a matrix $\mathbf{G}$ to represent the transmit filter bank, $\mathbf{G}=\left[g_{0,0},\quad ,g_{l,k}\right]\in{}^{N\times LK}$, and the same receive filter bank $\mathbf{Q}=\left[q_{0,0},\quad ,q_{l,k}\right]\in{}^{N\times LK}$ considers the L subcarriers and K symbols. For wireless transmission systems, transmission over a doubly selective channel can be modeled by employing a banded time-varying convolution matrix $\mathbf{H}\in\mathbb{C}^{N\times N}$, defined as $\left[\mathbf{H}\right]_{n,j}=h_{conv.}\left[n,n-j\right]$. The received data symbol vector can be expressed as:

$$\mathbf{y}=\mathbf{Q}^H\mathbf{HGx}+\mathbf{W}=\mathbf{Dx}+\mathbf{W} \tag{2}$$

where $\mathbf{x}=\left[x_{1,1},\quad ,x_{L,K}\right]\in{}^{LK\times 1}$ is the transmitted data symbol vector and $\mathbf{W}$ is the Gaussian noise.

Let $\mathbf{D}=\mathbf{Q}^{H}\mathbf{HG}\in{}^{LK\times LK}$ denote the transmission matrix of the FBMC/OQAM system.If the delay extension and Doppler extension are low enough, the transmission matrix can be rewritten as $\mathbf{D}\approx\mathrm{diag}\left\{\mathbf{Q}^{H}\mathbf{HG}\right\}\mathbf{Q}^{H}\mathbf{G}$.

3. LSMR EQUALIZATION ALGORITHM BASED ON DYNAMIC CONFIDENCE INTERVAL DETECTION

In an ideal static or narrowband channel, the transmission matrix $\mathbf{D}$ is usually a diagonal matrix, when there is no ISI or ICI interference. However, in dual-selective channels, due to the combined effect of time-selectivity and frequency-selectivity, the transmission matrix $\mathbf{D}$ becomes a sparse non-diagonal matrix. the diagonal elements of $\mathbf{D}$ represent the transmission gain of the signal at the current time-frequency position in the ideal case, corresponding to the useful signal components; the non-diagonal elements of $\mathbf{D}$ represent the mutual interference between the current time-frequency position and the other time-frequency positions, which is mainly reflected in inter-symbol interference (ISI) and inter-subcarrier interference (ICI). In this paper, to address the problem of high BER in dual-selective channels, an LSMR equalization algorithm based on Dynamic Confidence Interval Detection (LSMR-DCID) is proposed, which introduces dynamic confidence interval detection and ISI/ICI interference cancellation, and by exploiting the sparse property of the transmission matrix, the equalizer gets fast convergence.

3.1 LSMR equalization

According to the FBMC system transmission model, channel equalization can be modeled by solving the following least squares (LS) problem:

$$\min_{\hat{\mathbf{x}}\in\mathbb{C}^{LK\times1}}\left\|\mathbf{D}\hat{\mathbf{x}}-\mathbf{y}\right\|_{2} \tag{3}$$

The LSMR equalizer first generates α_{i+1}, β_{i+1}, $\mathbf{v}_{i+1}$, $\mathbf{w}_{i+1}$, using the Golub-Kahan bi-diagonalization decomposition[7]. α_{i+1}, β_{i+1} are chosen to normalize the orthogonal basis vectors $\mathbf{v}_{i+1}$ of the dataspace and $\mathbf{w}_{i+1}$ of the solution space so that their norms are equal to 1. The Krylov subspace is then constructed to solve the LS equation: $\mathbf{x}_i=\mathbf{w}_i g_i$, where g_i is obtained by solving for Eq.(4). Through several iterations, the equation solution $\mathbf{x}_i$ and residual $\mathbf{r}_i$ are updated.

$$\min_{g_i}\left\|\beta_1\alpha_1\mathbf{I}_1-\begin{pmatrix}\mathbf{B}_i^T\mathbf{B}_i^T\\ \alpha_{i+1}\beta_{i+1}\mathbf{I}_{i+1}^T\end{pmatrix}g_i\right\|_2 \tag{4}$$

The parameters α_1, β_1 and B_i can all be obtained by bi-diagonalization and paradigm operations on the transmission matrix $\mathbf{D}$, the residuals r. $\mathbf{I}_i$ is the i th column vector in the unit matrix.

3.2 Dynamic confidence interval detection and interference removal

In this subsection, an LSMR equalization algorithm based on dynamic confidence interval detection (LSMR-DCID) is proposed to take full advantage of the low complexity and high accuracy of the LSMR algorithm in solving linear least squares (LS) problems, while realizing the dynamic updating of the interference cancellation model. The core of the algorithm lies in the gradual optimization of signal quality through multi-stage iteration, and the specific process is shown in Algorithm 1.In the initialization phase, the observation vector and the FBMC transmission matrix are defined as: y and D. In the setting of the confidence region, the width of the initial confidence interval is defined as g, which is the constellation point threshold. the setting of g introduces an interval constant a. Taking the example of the 8PAM mapping data, defining the width of the interval between neighboring mapping data to be m, then:

$$g^{(0)}=\frac{m}{2}\left(1-\alpha\right) \tag{5}$$

Algorithm 1:LSMR equalization algorithm based on dynamic confidence interval detection

Input:transmission matrix $\mathbf{D} \in \mathbb{C}^{LK \times LK}$,observation vector $\mathbf{y} \in \mathbb{C}^{LK \times 1}$,Maximum number of iterations $k_{\max}$

Output: Equalized symbols $\hat{\mathbf{x}}$

Step1: Initialization: $\mathbf{D}^{(0)} = \mathbf{D}$, $\mathbf{y}^{(0)} = \mathbf{y}$,PAM Constellation Mapping $\mathbf{p}$,interval constant α ,Width of neighboring PAM symbols m ,Constellation point threshold $g^{(0)}$

Step2:iteration:

for $k = 1, 2, \cdots, k_{\max}$ do

a) Separation of real and imaginary parts: $\mathbf{D}_{eq}^{(k)} = \begin{bmatrix} \Re\{\mathbf{D}^{(k)}\} \\ \Im\{\mathbf{D}^{(k)}\} \end{bmatrix}$ $\mathbf{y}_{eq}^{(k)} = \begin{bmatrix} \Re\{\mathbf{y}^{(k)}\} \\ \Im\{\mathbf{y}^{(k)}\} \end{bmatrix}$

b) LSMR equalization: $\hat{\mathbf{x}}^{(k)} = \mathrm{LSMR}\left(\mathbf{D}_{eq}^{(k)}, \mathbf{y}_{eq}^{(k)}\right)$

c) Confidence interval judgment based on PAM mapping: Quantification error $r^{(k)} = \min\left|\hat{\mathbf{x}}^{(k)} - \mathbf{p}\right|$

if $r \le g^{(k)}$, PAM mapping, output to $\hat{\mathbf{x}}$

if $r > g^{(k)}$, Perform the next iteration

d) Status Update:

Constellation point threshold: $g^{(k)} = \frac{m}{2}\left(1 - \alpha\left(1 - k/k_{\max}\right)\right)$

transmission matrix: $\mathbf{D}^{(k)} = \mathbf{D}_U^{(k)}$

interference cancellation: $\mathbf{y}^{(k+1)} = \mathbf{y}^{(k)} - \mathbf{D}_R^{(k)}\mathbf{x}_R^{(k)}$

if $\mathbf{x}_U^{(k)}$ is empty，break out

end

Based on the consideration that values closer to the constellation point usually have higher reliability, this paper sets $\alpha = 0.8$, so as to focus more on adjudicating values far away from the constellation point. It effectively improves the system's judgment reliability in the high confidence interval around the constellation point. It also enhances the robustness to noise interference.

In FBMC/OQAM systems, the conventional complex-valued LSMR equalizer cannot efficiently estimate the transmitted data symbols because the equalizer includes both the imaginary part of the interference in the minimization problem, leading to a significant increase in the detection error. To solve this problem, the real and imaginary parts of the transmission matrix data $\mathbf{D}$ and observation vector $\mathbf{y}$ are extracted and stacked into a larger vector as inputs to the LSMR equalizer, respectively. The output signal $\hat{\mathbf{x}}^{(k)}$ after the LSMR equalizer is fed into the confidence interval judgment. Let $r^{(k)}$ be the quantization error of the k th iteration:

$$r^{(k)} = \min\left|\hat{\mathbf{x}}^{(k)} - \mathbf{p}\right| \tag{6}$$

Compare $r^{(k)}$ with the constellation point threshold $g^{(k)}$, which indicates the width of the credible interval.

If $r \le g^{(k)}$, the data falls into the credible interval and is output to $\hat{\mathbf{x}}$ after PAM mapping, otherwise the data falls into the untrustworthy interval and goes to the next iteration. After the judgment is completed, the data is divided into two parts: trusted interval data $\mathbf{x}_R^{(k)}$ and untrusted interval data $\mathbf{x}_U^{(k)}$.

There are three parameters updated in the state update phase, the first is the constellation point threshold $g^{(k)}$. After each iteration, the width of the trusted region will be gradually enlarged to $g^{(k)}$, which is finally equal to $m/2$. At this time, the untrustworthy interval is 0, which ensures the convergence of the whole iterative process. The second is the transmission matrix $\mathbf{D}^{(k)}$. The transmission matrix $\mathbf{D}_U^{(k)} \in \mathbb{C}^{LK \times q}$ corresponding to the untrustworthy interval data $\mathbf{x}_U^{(k)}$ is determined based on its index value, where q is the number of untrustworthy data. Thus the transmission matrix $\mathbf{D}_R^{(k)} \in \mathbb{C}^{LK \times (LK-q)}$ corresponding to the trusted interval data $\mathbf{x}_R^{(k)}$ is also determined for the reconstruction of the received signal and ISI suppression. Thirdly, the vector $\mathbf{y}^{(k+1)} = \mathbf{y}^{(k)} - \mathbf{D}_R^{(k)}\mathbf{x}_R^{(k)}$, where the received signal of the trusted interval is reconstructed as $\mathbf{D}_R^{(k)}\mathbf{x}_R^{(k)}$, is removed from the observed data, thus effectively eliminating the corresponding ISI and ICI. Subsequently, the updated vector $y^{(k+1)}$ will be used as an input to the next iteration of the equalization process.

4. NUMERICAL SIMULATION

In this section, the FBMC/OQAM system is built and the LSMR-DCID algorithm proposed in this subsection and the full-block MMSE, N-tap MMSE, interference cancellation[5], and single-tap equalization performance are simulated with the specific parameters listed in Table 1. In order to avoid the impact of channel estimation error on the detection performance, it is assumed that the receiver can fully obtain the channel response information. In this section, the convergence, complexity and bit error rate (BER) performance of the LSMR-DCID equalizer are analyzed, and the performance of the LSMR-DCID equalizer is compared with several typical equalizers.

Table 1. Algorithm simulation parameters.

parameter	value
carrier frequency f_c	2.5GHz
subcarrier spacing Δf	15 kHz
number of subcarriers L	24
number of FBMC symbols	30
number of OFDM symbols	14
modulation method	8PAM (64QAM)
filter	Hermite
filter length	$L_g = 4T$
emulation channel	VehicularA
travel speed	300km/h

4.1 Convergence

Figure 1 gives the number of unreliable symbols for the LSMR-DCID method with an input SNR of 20 dB and 8 PAM modulation (64 QAM) for different number of iterations, where the gray area indicates the range of reliable regions.When the number of iterations k < 5, more symbols are placed in the unreliable interval. With the increase of the number of iterations, the judgment threshold of the constellation point is gradually increased, i.e., the range of the reliable interval is gradually increased, the range of the unreliable region is gradually reduced, and the number of unreliable symbols is also significantly reduced. The data processed by LSMR equalization is finally quantized as reliable data. After 25 iterations, the number of unreliable symbols decreases to 0, close to zero, which fully verifies the good convergence of the algorithm.

4.2 Complexity theory

This subsection compares the complexity of full block MMSE equalization, N-tap MMSE equalization, iterative interference cancellation equalization, and LSMR-DCID. Full block MMSE equalization involves solving the inverse of a matrix of dimension $2LK \times 2LK$ with an arithmetic complexity of $\mathrm{O}\left(8L^3K^3\right)$. N-tap MMSE equalization takes into account only the proximity of the symbols, and the other symbols will be discarded as interferences, the arithmetic complexity is $O\left(8LK|N|^3\right)$ where $|N| \ll LK$. The complexity of the algorithm for the iterative interference cancellation The algorithmic complexity of equalization mainly comes from three aspects: the sparsity of the transmission matrix B, the number of iterations N_{iter}, and the inverse of the diagonal matrix, and the arithmetic complexity is $O\left(LKBN_{iter}\right)$. The complexity of LSMR-DCID relies on the number of nonzero elements in each row

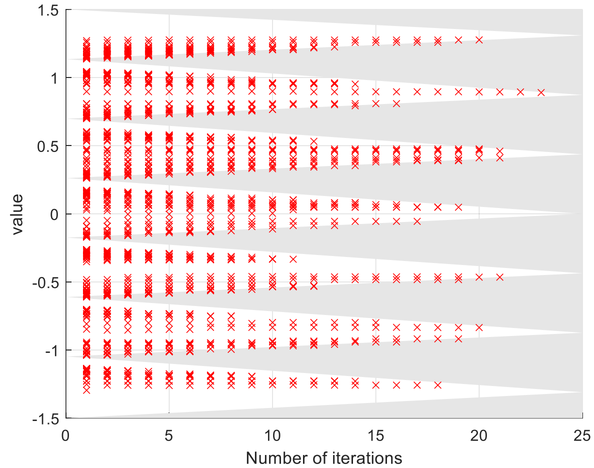

Figure 1. Distribution of the number of unreliable symbols for different number of iterations. ($SNR = 20\text{dB}$)

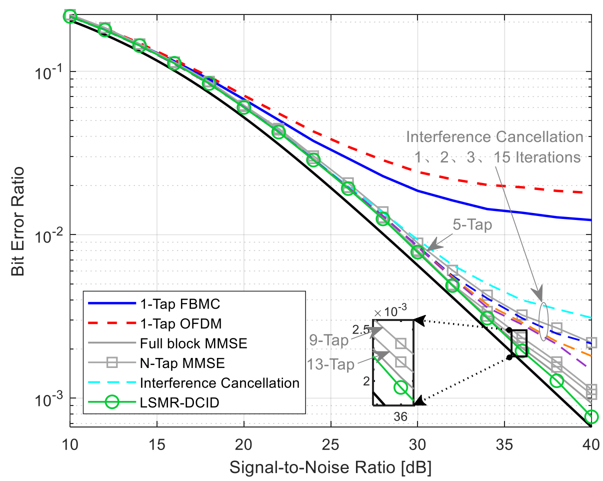

Figure 2. BER performance under different equalization methods.

or column of the transmission matrix B, the size of the constellation diagram Q, and the number of iterations K_{iter}. The arithmetic complexity of each iteration of the LSMR is $O\left(4LKBI\right)$ where I is the LSMR algorithm's number of iterations, and for K_{iter} iterations, the algorithm complexity is $O\left(4LKBIQK_{iter}\right)$.

4.3 BER

Figure 2 demonstrates BER performance comparison of different equalization methods under 64QAM modulation with the moving speed set to 300km/h. By analyzing the data in the figure, it can be concluded that the BER performance of all equalization algorithms gradually improves with the SNR. The figure presents the 5-tap MMSE equalization, 9-tap

MMSE equalization and 13-tap MMSE equalization of the FBMC system, respectively, which significantly reduces the BER compared to the first-order tap equalization, and the performance enhancement is more obvious especially in the middle and high SNR regions. The interference cancellation algorithm has better performance than 5-tap MMSE equalization after 2 iterations, but not as good as 9-tap MMSE equalization, and the figure shows that the BER is gradually reduced with the increase of iterations, and the system performance is still lower than 9-tap equalization after 15 iterations. The LSMR-DCID equalization algorithm proposed in this paper approximates the performance of full block MMSE equalization, and exhibits optimal BER performance over all tested SNR ranges. Its BER profile is consistently lower than other equalization methods, especially in the high SNR region, and is able to approach the theoretical bounds. Compared with other methods, the LSMR-DCID algorithm effectively improves the accuracy of symbol adjudication through the introduction of dynamic confidence intervals combined with the iterative optimization of interference cancellation, demonstrating high robustness and fast convergence.

5. CONCLUSION

In this paper, an LSMR equalization algorithm based on dynamic confidence interval detection is proposed. A channel equalizer based on the Least Squares Minimum Residual (LSMR) method is designed at the receiver side, and an iterative optimization strategy is proposed in combination with a dynamic confidence space symbol detection and interference canceller. By exploiting the sparsity of the channel matrix, the algorithm is able to achieve more efficient convergence. Simulation results verify the effectiveness of the proposed method, showing that the LSMR-DCID algorithm not only significantly improves the demodulation performance, but also possesses lower computational complexity in the time-frequency dual-selective channel environment. However, the LSMR-DCID algorithm is only validated in SISO/FBMC systems, and its application in MIMO/FBMC systems is a direction that requires in-depth research in the future.

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